GUIDE FOR TEACHERS

Mathematics Assessment Project

CLASSROOM CHALLENGES

Formative Assessment Lessons

A Brief Guide for teachers and administrators

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

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Mathematics Assessment Project

UNIVERSITY OF NOTTINGHAM & UC BERKELEY

The MAP *Classroom Challenges* Formative Assessment Lessons discussed in this guide are available, free-of-charge for noncommercial use, from the MAP website.

http://map.mathshell.org

The website also offers more details about MAP, summative assessment tasks, prototype professional development materials and details of the alignment of MAP materials with the Common Core State Standards.

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WHAT IS THE PURPOSE OF THIS BOOKLET?

This booklet is designed to help classroom teachers and district administrators understand what the MAP *Classroom Challenges* are and how they can help put the Common Core State Standards for Mathematics into action, including the Standards for Mathematical Practice. It describes the structure and organization of *Classroom Challenges* (CCs), how they address the CCSS, how they work[1], and how they function to stimulate and deepen students' mathematical thinking.

The booklet, along with the individual *Classroom Challenges* lesson and teacher's guides, describe a pedagogical stance and a role for the teacher that aligns with the research on formative assessment. Some teachers will likely find it different from their current practice. The booklet also includes the reactions of teachers who have used these lessons and their perspective based on interviews by an independent evaluation team with 12 middle and high school teachers. Each of the teachers interviewed piloted several *Classroom Challenges* in their classrooms. They come for three different states: California, Michigan and Rhode Island.

[1] The lessons have been developed, from research-based draft designs to the final products, through an iterative process of piloting and refinement. Reports on each pilot lesson are gathered from a trained observer, the teacher and the students – including samples of student work. This feedback covers how well (or not) the lessons are working as envisaged. These reports guide the revisions by the design team.

Summary introduction

WHAT IS A CLASSROOM CHALLENGE?

A *Classroom Challenge* (CC) is a classroom-ready lesson that supports *formative assessment*. The CCs help teachers assess and improve students' understanding of mathematical concepts and skills and their ability to use the "mathematical practices" described in the Common Core State Standards. And, they are available free of charge at: http://map.mathshell.org.uk/materials/lessons.php.

There are two types of CCs:

> ***Concept development lessons*** that are meant to first reveal students' prior knowledge, then develop students' understanding of important mathematical ideas, connecting concepts to other mathematical knowledge.
> ***Problem solving lessons*** are meant to assess, then develop, students' ability to apply their mathematical knowledge and reasoning in flexibly ways to non-routine, unstructured problems – within mathematics and with real world applications.

WHY USE CLASSROOM CHALLENGES?

Research has shown that formative assessment, as embodied in the *Classroom Challenges*, is a powerful way to improve student learning and performance. This approach first allows students to demonstrate their prior understandings and abilities in employing the mathematical practices, and then involves students in resolving their own difficulties and misconceptions through structured discussion. This results in more secure long-term learning, reducing the need for re-teaching that otherwise takes so much classroom time.

WHAT ARE THE KEY DIFFERENCES BETWEEN FORMATIVE ASSESSMENT TEACHING AND REGULAR TEACHING?

Formative assessment involves a change in "classroom culture" – with teachers and students moving into rather different roles from those common in most classrooms:

- Students take more responsibility for, their own work.
- Students engage in "productive struggle" with rich challenging tasks. Resolution comes only gradually through interactions and discussion in the lesson as students gain new facets of connected understanding.
- Students study fewer tasks, but in greater depth. They are asked to draft solutions, compare their approaches to others and redraft their ideas as a result of their discussions.
- The teachers' role is to prompt students to reflect and reason through their ideas. Teacher questioning is central to support students' thinking and depth of knowledge, and student growth. The teacher's role is not to provide answers and solutions. .

One way to shift practice, as described in the CCSS, is to begin by inserting occasional rich lessons such as *Classroom Challenges*. These lessons are designed to support these shifts with specific guides for each lesson. Teachers have found the Lesson Guides helpful and supportive in their efforts to implement formative assessment lessons and the CCSS mathematical practices and to expand their pedagogy and practice.

WHEN SHOULD I USE CLASSROOM CHALLENGES?

Classroom Challenges give powerful support for learning at various stages:

- Concept development CCs have been found to be helpful when used about two-thirds of the way into a unit of study, as they help teachers and students evaluate their learning, yet leave time to work on concepts that students are still struggling with, as well as time to build on the lesson's idea during the rest of the unit. Teachers have also used them at the start of a new school year or unit, when they are concerned or need more knowledge of their students' understanding of an important concept taught in an earlier grade. Using a formative assessment lesson, focused on that concept, can help determine students' level of understanding and identify gaps, allowing a teacher to be more focused in their instruction at the start of the new unit.
- Problem solving CCs are to be used every month or two throughout the year. They offer teachers the opportunity to develop students' proficiencies in the mathematical practices and concepts through deepening, consolidating concepts and building new connections.

The Structure of the Classroom Challenges

WHAT IS THE STRUCTURE OF THE CLASSROOM CHALLENGES?

MAP *Classroom Challenges* start with a short pre-assessment task, taken prior to the actual lesson. Teachers are asked to review their students' work so they can obtain an overview of the students' understanding and difficulties. The main lesson that follows uses a carefully structured sequence of individual work, small group discussions, and whole class sharing. At the end of the lesson, students take a post-assessment or engage in reflective activity, to help them realize what has been learned. While the sequence of activities is similar in the two types of *Classroom Challenges*, Concept Development and Problem Solving, their structure and purposes are rather different.

Concept development lessons focus on assessing and developing students' understanding of significant mathematical concepts, the interpretations that students have of the concepts and the connections between the concepts and their other knowledge. The Lesson Guide for each challenge supports the teacher throughout the lesson, with specific suggestions, sample questions, and examples for each phase of the lesson. They follow a format and flow that, with some variations, involves the following phases:

Prior to the lesson, a day or so before, the teacher assigns the pre-assessment that students complete individually. These typically take 10 to 15 minutes and are diagnostic, designed to reveal each student's understanding and misunderstandings of target concepts. The teacher reviews and analyzes the students' responses to gain an overview of the understandings and misunderstandings. Instead of scoring the papers, teachers are encouraged to create questions that will help students' reflect on specific issues they need to address. To assist teachers with this, a set of "Common issues" have been identified and included in the guide; each is linked to suggested follow-up questions teachers might give as feedback to their students.

The lesson has teachers engaging students in a related task, designed to expose their different ideas and ways of thinking. The tasks are rich and complex, allowing struggling students to gain access, while still providing challenges for the most capable. Students become aware of the inconsistencies in their own conceptions. This awakens a curiosity and desire to seek resolution through discussion. During this work, the teacher listens carefully to students and uses questioning (including the pre-prepared ones in the Lesson Guide) to promote deeper thinking and reflection. At various points, whole class discussions are used to share and resolve common difficulties. The lesson concludes by

sharing the different understandings and by generalizing and extending what has been learned. Students explain what they have done, and found.

After the Lesson students are given a post-assessment, similar to the pre-assessment, to demonstrate their learning from engaging in the concept lesson.

Problem solving lessons are designed to assess, then develop, students' capacity to apply their mathematical thinking flexibly to non-routine, unstructured problems, within math and from the real world – problems that students have not been taught how to solve. The challenge includes deciding on a strategy to solve the problem, applying the strategy, and then checking the solution to see if it works. The lesson design is built on using sample student work, which is provided. The students are asked to compare different approaches to a specific problem, understanding, critiquing, and learning from them. The lessons follow a format and flow that involves the following phases:

Prior to the lesson, a day or so before, the teacher assigns the task for students to complete individually. These typically take 10 to 15 minutes. The task is an unstructured non-routine problem, designed to reveal students' capabilities and limitations in problem solving. The task remains the focus throughout the lesson. The teacher assesses the students' responses to the task, noting their approaches to the problem and their difficulties. Teachers are encouraged to give feedback to their students by creating questions that will direct students' attention to strategies for problem solving. A set of "Common issues" related to the task, are given in the MAP Lesson Guide. Each issue is linked to suggested follow-up questions that teachers can use as feedback to their students. The teacher chooses or devises questions that direct students' attention to the strategies for problem solving.

The lesson: Students begin by individually reviewing their own solutions to the task in light of the questions raised by the teacher. After this reflection, students move into small groups to compare and build on their strategies for the problem. The teacher's role is to observe the groups, looking for different approaches and support their collective problem solving through questions like those suggested in the Lesson Guide.

The teacher facilitates the sharing of different approaches in a whole class discussion, with selected groups explaining their strategies. The students are then given some sample student responses for the same problem to discuss in their groups The sample responses show a range of approaches at different levels of completeness and mathematical sophistication. The work includes solutions using numbers, tables, graphs and/or algebraic reasoning. The students are asked to analyze and critique the work, compare solutions, and comment on their strengths and weaknesses.

After the lesson students work alone, again, to improve their individual solutions to the task and/or engage in a final reflection on what they have learned.

Timing. In both types of *Classroom Challenge*, the whole process takes the equivalent of at least two 55 to 60 minute class periods[2]. It is important to give students enough time to engage with the challenges involved, and to think them through, as deeply as possible. It is important that they are not rushed, and you should not be concerned if some students do not complete them. It is the learning that is important.

SOME REASONS FOR THE DESIGN OF THE CLASSROOM CHALLENGES

The pre-assessment phase reveals student thinking to the teacher. It is designed to surface individual's levels of understanding of the content and ability to engage in the mathematical practices. The work often reveals students' misconceptions about the mathematics. Having a clear, and often different view of students' abilities can help prepare a teacher to further the development of student learning in the

[2] Some teachers find the learning that is going on so impressive that they take more time.

remainder of the *Classroom Challenge*. The use of a pre-assessment, along with engaging in rich tasks, and working in groups, the instructional model used throughout the lessons, reflects the results of many research studies, which show that teaching practice that engages student reasoning and enables students to confront and resolve their own misconceptions through discussion, leads to more robust, long-term learning.

Many often ask, why, why don't students just remember the procedures they have been taught and practiced? The fact is, that approach works fine in the short term but, as every teacher knows, if procedural knowledge is not underpinned by conceptual understanding, students will quickly forget "how to do it". Research underlines this. For example, a detailed study of subtraction in arithmetic found that the students who made errors did so in many different ways – not too surprising. But the research *also* found that many students who got their calculations *correct* did so in different ways, not only the way they had been taught. Key research result:

> *Those who are effective at mathematics don't remember exactly how to do things; they remember roughly how, and can check and correct (debug) their own procedures.*

Implementation

Here we offer practical advice for teachers on how to use the *Classroom Challenges*.

GETTING STARTED – WHICH CC SHOULD I TRY FIRST?

The following CCs have been found to be good lessons for both teachers and students to start with. (CD = Concept development; PS = Problem Solving)

Grade 6 standards	Grade 7 standards	Grade 8 standards	High School standards
PS: Optimizing: Security Cameras	PS: Estimating: Counting Trees	PS: Modeling: Making Matchsticks	PS: Medical Testing
PS: Sharing Costs: Traveling to School	PS: Developing a Sense of Scale	PS: Solving Real-Life Problems: Baseball Jerseys	PS: Modeling: Having Kittens
CD: Laws of Arithmetic	CD: Evaluating Statements about Probability	CD: Solving Linear Equations in One Variable	CD: Manipulating Polynomials
CD: Mean, Median, Mode, and Range	CD: Steps to Solving Equations	CD: Interpreting Distance-Time Graphs	CD: Classifying Solutions to Systems of Equations

Yet, remember that the Common Core State Standards were designed as a *target,* representing "higher, fewer, clearer standards" than those common in current practice. Thus, teachers and districts should start with lessons that fit where their classrooms are now, even if an appropriate lesson is labeled for a lower grade. It will still be challenging.

To get the most from a *Classroom Challenge*, it is wise to use the lesson plan as presented a few times, before developing your own variations. The lessons have been carefully designed and written as well as trialed in multiple classrooms. At first the reasoning behind their design may not be apparent. It is also recommended that teachers first attempt with one of the lessons be with a class they are comfortable teaching. As you better understand the formative assessment process, you may use the CCs with all of your classes.

Top Ten Reasons for Using the Classroom Challenges

from the teachers who have used them

To learn about the benefits to students and teachers of implementing the Math Assessment Project (MAP) *Classroom Challenges*, Inverness Research interviewed twelve MAP pilot teachers. Their experiences of using *Classroom Challenges* (CCs) with their students were universally positive, while their views converged around ten key reasons for why other interested secondary mathematics teachers should consider using them too.[3]

#1 Students' understanding of the fundamental nature of mathematics expands and deepens when they engage in *MAP Classroom Challenges.*

> *The Challenges present the right portrayal of what mathematics is really about, so when students experience them, they see what the subject really is, how rich it is and how exciting it can be. I think it's important to help our students rethink what mathematics is, get them to understand that it's about problem solving and thinking critically, and that there is not just one way, one path. In fact this is one of the most important things the Classroom Challenges do for me -- they show students what mathematics is really about. For too long we have given them the impression that it is something else and that has been a disservice to them.*

#2 The *MAP Classroom Challenges* enable classroom teachers to enact the Common Core State Standards in Mathematics (CCSSM) and Standards for Mathematical Practice (CCSSMP) in their classrooms.

> *The Classroom Challenges really help a teacher understand what the Common Core is trying to get at, especially the Standards for Mathematical Practice, for example, practices like "making sense of problems" and "constructing viable arguments." I think CCs give teachers a better sense of what the Common Core is expecting and I think they will better prepare our students for the new line of assessment coming in the future.*

#3 The MAP *Classroom Challenges* engage and test students of all abilities, from those who have not previously participated in math classrooms to those who have consistently excelled.

> *One of the main benefits of the Classroom Challenges to students is that everyone has access. Because everyone is engaged and involved students don't feel stupid doing it. Even the students who are kind of hesitant don't feel shut down right away. They feel like they have something to say, everyone feels like they can contribute.*

#4 MAP *Classroom Challenges* demand that students talk and write about their mathematical understanding, and then analyze, synthesize, justify and often revise their thinking.

> *The CCs give students the chance to display and share their own thinking. Whether it be through formulas or drawings or notes or pictures or whatever it is, they are opportunities for students to explain how they got to a particular solution in their own mind. I think that is really beneficial for students.*

[3] Quotes are taken directly from interview transcripts, edited for grammatical correctness and readability. Inverness writes: "We are careful to maintain the integrity of quote; intent and meaning have not been altered."

#5 Student engagement increases, the "excitement level" in the classroom goes up and students learn more mathematics when teachers use *MAP Classroom Challenges*.

> *The Classroom Challenges are fun math. Students don't have to just sit there and listen to me, and that was very good for them ... instead the Challenges are on them. They have to work hard. They have to use all those problem-solving practices and higher level thinking skills. They have to make the connections, they have to figure it out, they have to take some chances, and that makes the math more relevant and ultimately deepens their conceptual understandings.*

#6 The *MAP Classroom Challenges* provide students learning experiences that "stress conceptual understanding of key ideas in mathematics," the major goal of the CCSSM.

> *Classroom Challenges are collaborative lessons built around a single concept … so CCs demand that students dig deeper, recall previous knowledge, collaborate with others and then really figure out where they are struggling … the focus is on developing understanding of that concept.*

#7 *MAP Classroom Challenges* help teachers shift from teacher-centered to student-directed classrooms, where responsibility for thinking and learning resides with students.

> *In my traditional classroom, I did all of the teaching. I was up front and I presented everything, but with the Classroom Challenges it was amazing to listen to the kids teach the other kids. Some of those students had a lot of knowledge and were able to pass it onto other kids. Witnessing that was a very valuable experience for me as a teacher. I didn't have to be in control all the time … I have changed the way I teach.*

#8 Using *MAP Classroom Challenges* allows teachers to hear and see their students in new ways, thereby illuminating what they know and can do mathematically.

> *The juiciest part of the CC lesson is when I see how students respond to the pre-assessment challenges. It gives you an honest look at where they are. So I would say that the learning that took place while the students were engaged in the assessment, and the learning that I did about their learning was much more valuable to me as a teacher than some of the other assessments that we were asked to give at my school.*

#9 The *MAP Classroom Challenges* are expertly designed and ready to use for teachers interested in implementing formative assessment lessons aligned to the Common Core in their classrooms.

> *I like the Classroom Challenges because they're already created. It's hard to be teacher, lesson designer and evaluator, so the benefit to me is being able to use a lesson that is totally mapped out, all of the materials are there … the CCs are all rolled up into one neat package.*

#10 MAP *Classroom Challenges* enhance teachers' instructional repertoire, enabling them to practice pedagogies such as active listening, questioning or facilitating small group discussions that promote deep mathematical learning.

> *I learned a lot about good mathematical questioning using the CCs. Now I feel like I have something that I am always paying attention to and asking myself about -- How do the designers frame this question? How am I going to get to this example? So I think using the Classroom Challenges has broadened my curriculum. And the Challenges have broadened me. They give me more tools for applying to my own lessons.*

Lesson preparation

FAMILIARIZE YOURSELF WITH THE TASKS

- Do the pre-assessment yourself. Try to anticipate the different approaches students may take and the difficulties they will encounter.
- Read the lesson plan through carefully, working the tasks and visualizing the sequence of activities for your students. Again, anticipate the different approaches students may take and the difficulties they will encounter.
- Prepare additional questions for use in the lesson, based on your understanding of where your students are and issues that are likely to arise. As usual with something quite new, lesson prep for the CCs will take more time than planning for your typical lesson.

ANTICIPATE WHAT WILL HAPPEN

- What are the big mathematical ideas?
- What might be conceptually difficult? What ideas will need emphasis through questioning?
- What questions might students ask? How will I respond and/or deal with these?
- What are the different strategies students may use?
- What kinds of question might I ask students that are stuck, or are using an incorrect strategy etc.?
- How can student-to-student discussion of the mathematics be encouraged?

PREPARE TIMINGS

- Note the timings in the Lesson Guide and plan how to match them with the length of your class periods. NOTE: Initially, most teachers find they need more time than is suggested.

EVALUATE THE PRE-ASSESSMENT RESPONSES

- Read through the Common Issues table in the Lesson Guide. It gives information about common issues and misconceptions to look for in student work, based on classroom trials and prior research.
- Look through your students' responses. Do not score them; look for misunderstandings and different approaches.
- Decide how you are going to give students feedback. You may choose to:
  - Write comments on each paper. This takes time but is worth doing occasionally (Unlike scoring, it leads students to think about their reasoning.)
  - Create and make copies of a list of common questions and comments regarding the student work and highlight for each student those that you think he or she should pay attention to in the light of issues in their response.
  - Use the questions in the common issues table, projecting the questions and talking through the misconceptions.

MANAGING THE MAIN LESSON

Each lesson has its own guide. The Lesson Guide offers teachers detailed outline and support. Teachers are encouraged to adhere to the guide, including the timings, as much as possible. It is based on feedback from the teachers and classroom observations in the trials.

REFLECTING ON WHAT HAPPENED

Teachers are encourage to reflect on the learning experience after using a lesson, such as:

- What went well with the lesson?
- Did the lesson go as envisioned? If not, why?
- What evidence was there of student learning?
- How did the students respond, in their attitudes and their discussion?
- What will you do differently next time? Why?
- How might the structure and pedagogy of the CC lesson carry over to other lessons?

Classroom culture: student and teacher roles

In every classroom, there is a set of mutual expectations of what each will do. The MAP *Classroom Challenges* involve a classroom culture that is somewhat different from most teaching, with a different set of roles and expectations for students and the teacher.

Many classrooms follow a typical format. The teacher is expected to manage the classroom, explain new concepts and procedures, demonstrate these through worked examples, assign exercise problems that are similar, and ask questions to check students' answers. Students will follow the explanations, work on the problems in the way they have been shown, and answer a question as well as they can – usually in a few words. The teacher will know the (single) correct answer, and tell students if theirs is correct. If students are not "getting it", the teacher will explain again.

In the *Classroom Challenges*, the roles are different. Students are expected to take more responsibility for their own learning. The teacher, while managing the classroom and setting tasks, works mainly through questions designed to help students to clarify and improve their own reasoning. In discussion with others, students compare approaches and strategies and check their own and each others' solutions. The teacher focuses discussion on reasoning rather than just answers – questions of *why?* not just *what or how?,* expecting responses in complete sentences, featuring the word *because*.

It is helpful to think of this changed classroom culture holistically, as *moving students into teacher roles*. In the *Classroom Challenges*, students take on some of the traditional *directive roles*: managing their own learning, explaining, creating tasks. Teachers move into less directive, more *facilitative roles* in the learning process: challenging students about their thinking (*what?* and *why?*); listening and observing; discussing a student's reasoning as a fellow student might, rather than as "the expert"; providing information (not answers), but only when asked.

The Lesson Guides for the *Classroom Challenges* reflect these changes in the context of that lesson. Research shows that this formative assessment approach is more effective in promoting learning – not so much at the end of the lesson or a unit but in the long term, months and years later. If you are interested in knowing more about this, go to the MAP website: http://map.mathshell.org.uk/materials/lessons.php. .

THE STUDENT'S ROLE

Students are expected to:

- Make sense of problems and persevere in solving them, look for and plan solution paths (MP1)
- Understand the approaches of others (MP1)
- Make sense of quantities and their relationships in problem situations (MP2)
- Attend to the meaning of quantities, not just how to compute them (MP2)
- Understand the approaches of others, communicate to others, and respond to the arguments of others (MP3)
- Try to communicate precisely to others (MP6)
- Justify their conclusions (MP3)
- Continually ask themselves, "Does this make sense?" (MP1)

THE TEACHER'S ROLE

The teacher is asked to create the opportunity for students to dig in and make sense of some mathematics, to facilitate student learning and support students by:

- Giving students time for "productive struggle" with each task
- Questioning, rather than explaining the math (explaining what you want them to do but not how to approach it)
- Asking for explanations, not answers. *(Tell us how you got that. Why does that make sense?* and, asking again, *Can you say more about that?*)
- Working to implement the *Classroom Challenge* as written

…and avoiding:

- "Scaffolding" – breaking up a task into steps (any scaffolding in the CCs has been carefully designed and developed)
- "Clearing up misunderstandings" during a CC – this is assessment
- Letting an activity run long past the suggested time (unless you see the students are playing the roles listed above in a particularly productive way).

CRITICAL POINTS

In teaching a *Classroom Challenge*, it is good to be aware of places where common teaching moves will undermine the challenge. For example:

- When you ask a question of a small group, don't look for a quick response but leave it with them. It takes thinking time to create a worthwhile response to a good question.
- Don't do "mini-lessons" that try to "fix" students' misconceptions and wrong moves so that they get a "right answer". Instead, ask questions that help students think more about their work and assumptions. This helps *them* make progress with the task, and more importantly with the content.
- Don't *evaluate* student responses, in words or looks; keep *them* responsible for that. Instead, ask a further question like: *Why do you think so?* or ask another student.

In a concept development CC:

- Note the structure, timing and suggested focus of the introduction. Its goal is to foreground the misunderstandings, *not resolve them* – that is for the discussions that follow in the collaborative work time and the last whole class discussion.
- Group students such that all students will have an opportunity to engage in the activity and share their thinking.
- The whole group discussion at the end of the lesson is the most important part of the lesson and critical for student learning.

In a problem solving CC:

- It is important for the students to engage with the task holistically – if you need to clarify it, perhaps for students with reading difficulties, make sure you don't break it into steps or smaller tasks.
- Avoid problem specific suggestions or questions. Keep to strategic questions: *What have you done so far (and why)? What have you tried?*
- If necessary after a while, ask questions related to problem solving strategies: *Have you looked at some simple examples? Have you organized what you know, and what else you need to find out? How might you represent this with mathematics? Do you see any patterns?*
- Group students who may have different approaches to solving problems.
- The whole class discussion at the end is important for student learning.

These things, if new to your teaching style, will gradually become natural as you and your students change the classroom culture and build new and different habits of mind and dispositions for doing mathematics.

Design principles and design features

Malcolm Swan, the lead designer for MAP, has developed the design principles[4] used for the *Classroom Challenge* lessons, based on 30 years of design research at the Shell Centre and elsewhere.

Teaching design for conceptual understanding *is more effective when we:*

- ***Use rich, collaborative tasks****. The tasks we use should be accessible, extendable, encourage decision-making, promote discussion, encourage creativity, encourage 'what if' and 'what if not?' questions. Students should not need to start or finish at the same point, enabling everyone to engage with the activity.*
- ***Develop mathematical language through communicative activities.*** *Mathematics is a language that enables us to describe and model situations, think logically, frame and sustain arguments and communicate ideas with precision. Students do not know mathematics until they can 'speak' it. Interpretations for concepts remain mere 'shadows' unless they are articulated through language. We find that many students have never had much opportunity to articulate their understanding publicly.*
- ***Build on the knowledge learners already have.*** *This means developing formative assessment techniques so that we may adapt our teaching to accommodate learning needs. Lessons do not follow the traditional pattern for explanation followed by exercise. Instead, the teacher asks questions which expose and assess existing ways of thinking and reasoning before explaining. The teacher listens to the discussions before joining in, then attempts to prompt students to articulate their thinking and reasoning. Teacher explanation follows this discussion, it does not pre-empt it.*
- ***Confront difficulties rather than seeks to avoid or pre-empt them.*** *Effective teaching challenges learners and has high expectations of them. It does not seek to 'smooth the path' but creates realistic obstacles to be overcome. Confidence, persistence and learning are not attained through repeating successes, but by struggling with difficulties. Conceptual obstacles are part of design, deliberately included to provoke discussion.*
- ***Expose and discuss common misconceptions and other surprising phenomena.*** *Learning activities should expose current thinking, create 'tensions' by confronting learners with inconsistencies and surprises, and allow opportunities for resolution through discussion. The activities encourage misconceptions and alternative interpretations to surface so that they may be discussed. Conflicts originate both internally, within the individual, and externally, from an individual's interpretation of another person's alternative viewpoint.*
- ***Use higher-order questions.*** *Questioning is more effective when it promotes explanation, application and synthesis rather than mere recall. Teachers are encouraged to prompt students to reflect and explain through the use of open prompts that begin " Explain why..."; "Show me an example of.."; "How do you know that...?"*
- ***Make appropriate use of whole class interactive teaching,*** *individual work and cooperative small group work Collaborative group work is more effective after learners have been given an opportunity for individual reflection. Activities are more effective when they encourage critical, constructive discussion, rather than argumentation or uncritical acceptance. Shared goals and group accountability are important. Teachers are advised to gradually establish 'ground rules' for discussion among students and then behave in ways that encourage dialogic and exploratory talk.*
- ***Encourage reasoning rather than 'answer getting'.*** *Often, learners are more concerned with what they have 'done' than with what they have learned. Aim for depth rather than for superficial 'coverage', telling students that comprehension is more important than completion. The teacher's role is to prompt deeper reasoning by asking students to explain, extend and generalize.*

[4] see http://www.educationaldesigner.org/ed/volume1/issue1/article3/index.htm

- ***Create connections between topics both within and beyond mathematics.*** *Learners often find it difficult to generalise and transfer their learning to other topics and contexts. Related concepts remain unconnected. Effective teachers build bridges between ideas, so the design is multiple connections between different representations.*
- ***Recognize both what has been learned and also how it has been learned.*** *What is to be learned cannot always be stated prior to the learning experience. After a learning event, however, it is important to reflect on the learning that has taken place, making this as explicit and memorable as possible. Allow students to share their findings through the public display of their work. Encourage students to extend and generalize their ideas by making small changes to the examples, and then to explicitly formulate rules for equivalence. This helps the teacher recognize and value the contributions of students, extending and institutionalizing them.*

SOME DESIGN FEATURES AND THEIR PURPOSES

Teachers may find it useful to understand some of the design tactics used in the *Classroom Challenge* Lessons.

Posters and felt pens

Familiar in the elementary grades and some middle school classrooms, the use of posters in group work may be new to many high school teachers. Used in small groups, they

- provide a medium for students to work and discuss the mathematics together, and to create a common solution.
- allow the teacher to observe the work without entering the group's space.

They also help the groups to present their work on the classroom wall for comparison and comment by other groups.

NOTE: Make sure students' poster work focuses on the mathematics – not on their artistic talents!

Card sorting and matching

These activities allow the students in each group to try out their ideas, generating hypotheses for the group to discuss, critique and improve.

In gluing down their selections on their poster, the group makes a considered statement for the teacher (and/or other groups) to critique.

This learning process makes the complications of cutting out cards worthwhile.

Samples of student work

Used mostly in problem solving lessons, these ask students to consider in detail a variety of approaches. None of the solutions is complete and correct, so that students can be asked to *critique* each one; this is a much more active and engaging process than simply *understanding* them. It puts the student on the same level as those who composed the solutions.

Carefully selected, some of the samples will use up-to-grade mathematics – more advanced than is likely to have been used within a typical class[5]. This shows the students how such mathematics can give more powerful solutions (e.g. solving a whole class of problems).

[5] People cannot use mathematics they have been taught recently in solving non-routine, complex problems. The challenge of such tasks lies in deploying the mathematical practices effectively, choosing and planning an approach and reasoning it through – this requires concepts and skills that have been well-integrated into the student's connected mathematical understanding.

Differentiation by support

Do not be tempted to change the tasks, give all students the opportunity to attempt the same task but offer different levels of support, depending on needs that become apparent. This avoids the danger of prejudging learners, lowering their and your expectations.

Common issues

The Common Issues table is an important resource for teachers. It is based on common difficulties observed in research and in trials of the lesson. The questions and prompts can be used:

- to help students progress in their work after the pre-assessment, and throughout the lesson.
- to support students who are struggling, or using an incorrect strategy
- as a tool to understand student thinking.

Lesson Genres and the Mathematical Practices

Throughout this booklet, we have described the similarities and differences between problem solving lessons and concept development lessons. Among the concept development lessons there are four distinct genres, each of which emphasizes particular mathematical practices[6].

TRANSLATION: LINKING MULTIPLE REPRESENTATIONS

What is another way of showing this?

Representation is at the heart of problem solving in mathematics (MP1), and for modelling (MP4) real world situations. It is important to be able to "translate" information between different more-or-less abstract representations (MP2) – words, pictures, numbers, tables, graphs and symbolic algebra.

The Grade 8 lesson *Interpreting Distance-Time Graphs* is an example of this genre.

TESTING ASSERTIONS & "MISCONCEPTIONS"

Always, Sometimes or Never True?

Students are frequently confused about the status of a mathematical statement. For example, what is the difference between *factoring a quadratic* and *solving a quadratic equation*? Yet understanding under what circumstances a statement is true is at the heart of mathematical reasoning (MP2) and thinking with precision (MP6)

The High School lesson *Evaluating Statements about Length and Area* is an example.

CLASSIFYING OBJECTS & CHALLENGING DEFINITIONS

What is the same and what is different?

Classifying is at the heart of recognising and making use of attributes and structure (MP7) and developing and critiquing definitions. (MP6) Students must learn to discriminate carefully, developing and testing categories and definitions. *I can't find a case with none these properties; can I find a reason why?*

The Grade 8 lesson *Classifying Solutions to Systems of Equations* is an example.

[6] Each problem-solving lesson typically involves many of the practices but, because of the higher strategic "load" in non-routine tasks, they necessarily involve more elementary concepts and skills.

MODIFYING PROBLEMS & EXPLORING STRUCTURE

What happens if I change this?
How will it affect this?

Mathematics is about generalization and abstraction – the same tools can be useful in tackling a great variety of problems. (MP2, MP7 and MP8) Students learn by exploring the effects of varying the elements of a problem, exploring how one variable depends on another, and by creating their own problems in mathematics or through modelling (MP4).

The Grade 8 lesson *Modelling Situations with Linear Equations* is an example.

ON TECHNOLOGY

You will have seen throughout this booklet, that all the classroom challenges emphasize reasoning, making and critiquing arguments (MP2 and MP3) and involve problem solving and modelling (MP1 and MP4) to some extent. To ensure they are usable in every class, none of the lessons *requires* technology but, where it is available, there are many opportunities to choose and use it appropriately and effectively (MP5).